Design of a MATLAB-based Teaching Tool in Introductory Fractional-Order Systems and Controls

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***Abstract*—In this work-in-progress paper, the topic of fractional-order modeling and control in education is discussed. The study of fractional calculus, fractional-order systems (FOS) and the applications thereof have been of interest to many scientists and engineers as evidenced in the recent exponential growth in the number of corresponding research publications worldwide. One particular area of study is fractional-order control design due to the additional flexibility in tuning the controllers that fractional-order calculus provides. Although there is a wide growth in research productivity in this particular area, the number of educational resources are still limited while researchers and students would find their own respective ways to understand the concepts of fractional calculus and how fractional-order systems and controls work. The goal of this contribution is to establish a simple but effective set of teaching tools that can be used by students and researchers in understanding concepts of fractional calculus and its applications, especially on FOS and more specifically on control design. The MATLAB tool currently being developed is a graphical user interface (GUI) that is designed to show the time and frequency responses of FOS models and the primary function of which is to study basics of fractional PID controller design. The presented tool is a potential educational material that can be used either in an undergraduate or graduate class in fractional calculus applications, or could be used as a tool for self-learning by those who are interested in the topic.**

***Index Terms*—Fractional Calculus; Control Design; Educational Software; Simulation; Learning Objectives, Concept Inventory**

I. Introduction

Fractional-order calculus or, equivalently, *noninteger calculus* referring to the orders of differentiation or integration, has been the object of discussion and development for more than 300 years. However, there is still no general intuitive explanation for the meaning of a noninteger derivative similar to that of its classical counterpart [1]. For this reason, coupled with the fact that fractional-order systems are much more complex in nature, not to mention some additional issues with theoretical aspects of the discipline [2], [3], it has not been a part of standard mathematics curriculum at large, nor has it seen widespread use in applied science subjects taught in universities.

It has been demonstrated [4] that applying fractional calculus in various scientific fields, including automatic control systems [5], leads to considerable benefits in terms of modeling accuracy and additional flexibility in control design [6]. Meanwhile, system theory and control system design are applied in numerous engineering fields [7]. As a consequence, the interest towards FOC increased, as did the number of publications related to FO modeling and control. In this specific area, MATLAB environment [8] is oftentimes used as a de facto standard tool for teaching control engineering [9], as well as doing related research. For achieving specific goals, various MATLAB toolboxes are used, such as the acclaimed Control System toolbox. There is still no official toolbox for MATLAB that includes features necessary to implement FO modeling and control, however. Therefore, several independent packages exist that provide the necessary functionality for working with FO systems and controllers. Among more complete ones are CRONE toolbox [10], Ninteger [11], and more recently FOMCON toolbox [12] and specific MATLAB tools [13], the last two developed by the authors of this paper.

The aim and contribution of this paper is twofold. First, the current methodology for providing instruction related to fractional-order modeling in control in the context of an advanced control curriculum is described and a concept inventory [14] is proposed. Based on the feedback and data received from students, we then describe the work-in-progress contribution—A MATLAB-based tool—the first in the prospective series—leveraging the functionality of the present MATLAB tools that is designed to assist students in intuitively understanding the effects and corresponding benefits of fractional-order systems and controllers.

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It is assumed that the reader has basic knowledge of system theory and control systems. The structure of the paper is as follows. In Section II the mathematical context for fractional-order modeling and control is set to illustrate the studied problems. In Section III the teaching tools are presented, concept inventory is proposed, and the newly developed MATLAB tool described. Then, in Section IV an example are provided pertaining to the intended use of the tool. Finally, conclusions and future research and development options are drawn in Section V.

II. Problem Overview

A. Modeling

The main idea of fractional-order calculus is the generalization of integration and differentiation to a non-integer order operator ${}_a\mathscr{D}_t^\alpha$, where a and t denote the limits of the operation and α denotes the fractional order such that

$$ {}_a\mathscr{D}_t^\alpha = \begin{cases} \frac{\mathrm{d}^\alpha}{\mathrm{d}t^\alpha} & \Re(\alpha) > 0, \\ 1 & \Re(\alpha) = 0, \\ \int_a^t (\mathrm{d}\tau)^{-\alpha} & \Re(\alpha) < 0, \end{cases} \tag{1} $$

where generally it is assumed that $\alpha \in \mathbb{R}$, however the order can also be a complex number. Based on this definition, one can construct fractional-order differential equations (FODEs) that can be used to describe linear, time-invariant, single input, single output dynamic continuous time system models:

$$ \begin{aligned} &a_n \mathscr{D}^{\alpha_n} y(t) + a_{n-1}\mathscr{D}^{\alpha_{n-1}} y(t) + \cdots + a_0 \mathscr{D}^{\alpha_0} y(t) = \\ &\quad b_m \mathscr{D}^{\beta_m} u(t) + b_{m-1}\mathscr{D}^{\beta_{m-1}} u(t) + \cdots + b_0 \mathscr{D}^{\beta_0} u(t), \end{aligned} \tag{2} $$

where $(a_i, b_j) \in \mathbb{R}^2$ and $(\alpha_i, \beta_j) \in \mathbb{R}_+^2$. The system is said to be of *commensurate-order* if in (2) all the orders of derivation are integer multiples of a base order q such that $\alpha_k, \beta_k = kq, q \in \mathbb{R}_+$. In the field of system theory and control system design it is more common to study dynamic systems described by (2) in the Laplace domain [7]. The corresponding fractional-order transfer function representation of a process model including an additional input delay term given in the time domain as $u(t) = u_d(t - L)$ is

$$ G(s) = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \cdots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \cdots + a_0 s^{\alpha_0}} \mathrm{e}^{-Ls}, \tag{3} $$

where it is usual to take $\beta_0 = \alpha_0 = 0$ so that the static gain of the system is given by $K = b_0/a_0$,and $L \in \mathbb{R}_+$.

Several methods exist to study the behavior of the systems described by (3) in the time domain, i.e., based on the Mittag-Leffler function that generalizes the concept of the exponential function found in solutions of classical differential equations. However, deriving and using an analytical solution of FODEs is often complicated and impractical, therefore numerical methods based on, e.g., the Grünwald-Letnikov definition of the FO operator are used [6]. In addition, using approximations of (3), e.g., the Oustaloup filter [10] or the refined version thereof [15], is common practice, especially for purposes of real-life implementation [16]. Still, it is more convenient to study the behavior of FO systems in the frequency domain since to obtain the corresponding response it is enough to set $s = j\omega$ in (3) and do a frequency sweep for $\omega \in [\omega_{min}, \omega_{max}]$ obtaining the magnitude and phase angle response for every frequency point. Moreover, frequency domain analysis of FO systems is more intuitive due to simple relations between the frequency characteristics and orders of differentiation and/or integration. This is why it is very important to establish general relations between frequency domain and time domain analysis especially for the purpose of control design, the very problem we attempt to solve with this contribution.

B. Control Design

In what follows, we consider the classical negative unity feedback control loop shown in Fig. 1.

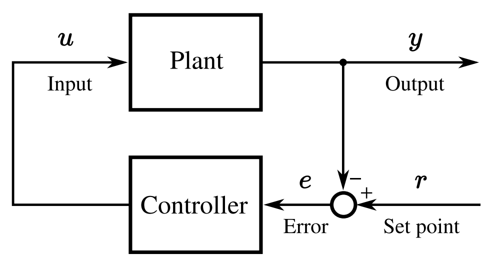
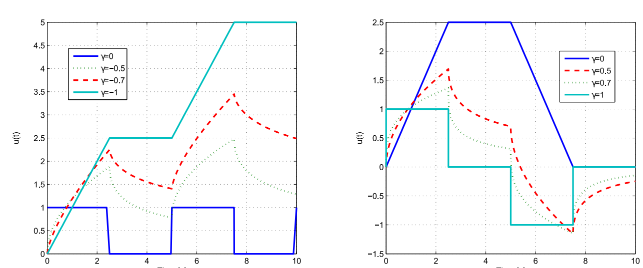


Fig. 1. Negative unity feedback system

Here, the objective of the control system is to manipulate the plant input u via the controller to minimize the error e, i.e., difference between the desired output r (reference value) and the true output of the plant y, i.e., we consider the *output tracking* problem. In real-life industrial applications a proportional-integral-derivative (PID) controller is typically used. The fractional-order PID controller (FOPID) is an extension of the conventional PID controller [17] such that

$$ C(s) = K_p + K_i s^{-\lambda} + K_d s^{\mu}, \tag{4} $$

where λ and μ are orders of integration and differentiation, respectively. To illustrate the additional dynamic flexibility of this controller Figs. 2a and 2b show the time domain responses of a fractional component s^γ, $\gamma \in [-1, 1]$ under a square- and trapezoidal-shaped input signal, respectively.



(a) Fractional integrator

(b) Differentiator

Fig. 2. Control actions in the time domain corresponding to s^γ

In the frequency domain, the variation of γ translates to the following:

- A constant change in the slope of the magnitude curve that varies between -20γ dB/dec and 20γ dB/dec.
- A constant delay in the phase plot that varies between $-\pi\gamma/2$ rad and $\pi\gamma/2$ rad.

Based on frequency domain characteristics, some control system quality measures can be derived. We consider the following specifications depicted in Fig. 3:

- *Gain margin* G_m and *phase margin* φ_m specifications—these are relative stability and robustness measures in control systems;
- *Robustness to plant gain variations*: a flat phase of the system is desired within a region of the system critical frequency ω_c; this is an important measure because the parameters of all real-life systems drift away from their nominal values and this specification ensures that this does not affect the performance of the control system.

The key aspect here is that although achieving these specifications is possible with a conventional PID controller, its fractional counterpart provides two additional degrees of freedom, so more specifications can be satisfied at the same time. Moreover, only a FOPID controller can properly compensate fractional-order dynamics of the plant.

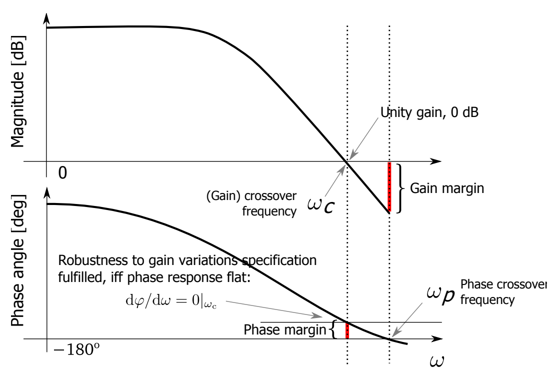


Fig. 3. Frequency domain specifications

The main problem is thus developing an intuitive MATLAB tool to illustrate all specifics of FO system analysis and control design described above to be used in instruction activities adhering to a concept inventory proposed in the following section.

III. Methodology

A. Previous Results

The following discussion is based on the analysis of several years of teaching experience related to advanced control courses read in Tallinn University of Technology and Technical University of Kosice. The courses deal with several topics, including fractional-order modeling and control. In this case, it is assumed that the students have previous experience in the design of linear control systems. One particular learning objective is to teach *model based control design* in the context of FO modeling and has the following components:

1) Illustration of the specifics of fractional-order systems and controllers through time- and frequency domain simulations and analysis of the obtained results.
2) Identification of a fractional-order dynamic system and assessment of the quality of the obtained model.
3) Design of a suitable FOPID controller of the obtained model and assessment of its performance on a simulated close-to-real-life plant.

The whole procedure is intended to show real-life application of FO systems and controls. Every part of the procedure is carried out during a lab exercise in the MATLAB/Simulink environment using specific supporting tools [13], [18]. Currently, the tools provided in [12] allow to tune a FOPID controller for the identified model using optimization based on time- and frequency domain specifications via a graphical user interface. However, this approach does not provide clear insight into the inner workings of the tuning algorithm, nor does it provide means to intuitively understand the meaning and effect of each particular parameter of the controller on the overall system. The tools in [13] on the other hand also extend several concepts for nonlinear control systems, but are not yet supported by corresponding graphical user interfaces.

Table I shows the student grade distribution in the courses in Tallinn University of Technology (TUT) and Technical University of Kosice (TUKE) for the 2016–2017 academic year. On average, 39 students participate in the courses. Setting aside some general differences in teaching methods, we focus on the fact, that in TUT students generally achieve higher grades, but also there are more students who fail to pass the course. However, this distribution has changed over the last four years before reaching the current state. For solving the FO modeling and control exam problem, FOMCON toolbox has always been used, and it has also seen significant improvement during this time, especially in terms of additional graphical user interfaces and improvement of user experience. Based on this fact, we choose to proceed with the design of additional graphical user interfaces capturing the essentials and advanced topics in FO modeling and control. To support the main learning objective and based on the above discussion, the following concept inventory [14] is proposed:

TABLE I
Percentage of students achieving certain grades

University	A	B	C	D	E	FX†
TUT	49%	29%	7%	0%	0%	15%
TUKE	3%	35%	30%	16%	14%	3%

†—failed to pass the course

- Specifics of FO dynamics: strong dependence on memory; self-similarity; slow convergence;
- Specifics of FOPID controller compared to conventional PID controller; understanding of fractional control actions;
- Frequency domain specifications, stability, and robustness in case of FO systems;

- The dependence of frequency domain characteristics on the orders of integration/differentiation and the relation of these characteristics with the time domain response;
- Tuning FOPID controllers via linear approximations of nonlinear plants; using linear FOPID controllers for control of nonlinear systems.

In this work, we describe the first solution out of a planned series—a MATLAB and FOMCON toolbox based graphical user interface for studying the relationship between the time- and frequency domain performance of the control system.

B. Proposed Work-in-Progress FOPID Tuning Tool

The designed FOPID controller tuning tool [19] is depicted in Fig. 4. The tool requires FOMCON toolbox to function

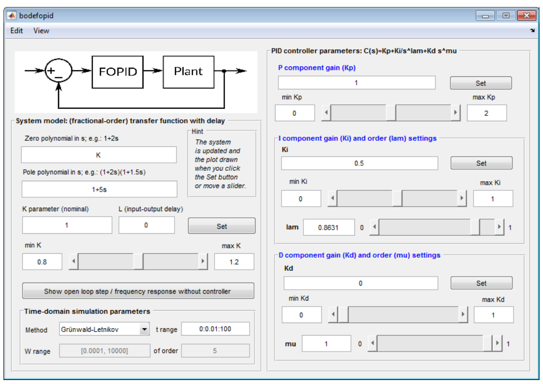


Fig. 4. Graphical user interface of the BODEFOPID tool

and has the following features that aim to solve the outlined problems:

- **Key feature**: all FOPID parameters have a slider, so it is very easy to test how a change in a single parameter reflects on both time- and frequency domain characteristics of the control system; any change in the slider triggers a new simulation and updates the graph with the results (see an example in Fig. 5).
- **Key feature**: Every change in parameters is logged and the results are displayed in MATLAB command line. The previous response is always plotted on the graph alongside the new response. In addition, if required, each time a simulation occurs, all relevant parameters are saved into MATLAB workspace. Thus, it is possible to reuse these parameters in Simulink to simulate the work of the controller with a nonlinear plant.
- For plant entry, a symbolic parser is used which facilitates entry of complex FO models.
- Open-loop response of the plant alone can be simulated separately to determine the type of compensator needed in the closed loop.
- The plant is parametrized in one parameter which by default is static gain—this can be used to test the robustness to gain variations specification.

Consider an example that showcases the usage of the tool.

IV. Example

Let the plant to be controlled be described by a simple integer-order model

$$G(s) = \frac{K}{1+5s}, \tag{5}$$

where $K \in [0.8, 1.2]$ with a nominal value of 1. The task is to design a suitable controller for it. One possible workflow using the tool described previously is as follows:

1) Set the plant parameters, simulate open loop response.
2) Start by adding an integral component $K_i = 0.5$. Simulate closed loop response.
3) Using the λ-slider, decrease the value of the order of integration. The results are updated automatically.

The final settings are shown in Fig. 4, while the comparison of control system performance is shown in Fig. 5. It can be seen that the slope of the magnitude of response changed, which resulted in a different time domain response where there is no longer an overshoot. The value of the phase margin changed from $\varphi_m = 64.62°$ at $\omega_c = 0.3162$rad/s to $\varphi'_m = 76.70°$ at $\omega'_c = 0.3126$rad/s which is reflected in MATLAB output.

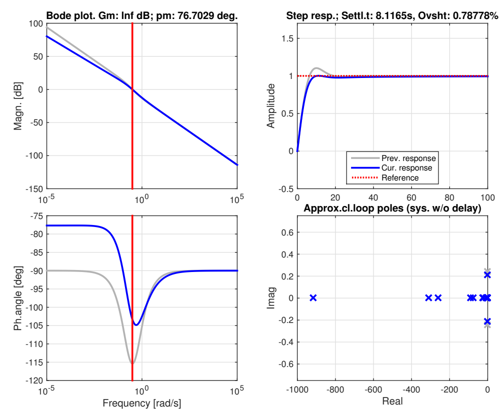


Fig. 5. Example BODEFOPID graph plotted in response to a user action

V. Conclusions

In the present paper, we have outlined a particular problem related to the learning objective of an advanced control course having to do with fractional-order modeling and control. The previous treatment of the problem was described, and a work-in-progress tool adhering to a certain concept inventory was proposed along with a relevant usage example. Future work will be focused on integrating this tool into the course and formal analysis of the results of this integration.

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